Output-sensitive Complexity of Multiobjective Combinatorial Optimization

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In multi-objective optimization, several objectives are optimized simultaneously, i.e. each solution has an objective vector instead of a single objective value. We try to find solutions that cannot be improved in one objective without worsening it in another objective, these are called *Pareto optimal* or *efficient* solutions and their objective vectors are called *non-dominated*. While in single-objective optimization there only exists (at most) one optimal objective value, several solutions with different non-dominated vectors can exist, which form the *Pareto front*. The aim of a multi-objective optimization problem usually is to find the whole Pareto front.

For many combinatorial problems, which are polynomially sovable in the single-objective case, there exist exponentially many non-dominated objective vectors. Hence, unless $P = NP$, the Pareto front can not be computed in exponential time. Even though the problem is then clearly NP-hard, it remains unclear, if its complexity of the problem only depends on the size of the output set or if it is inherently hard. Therefore, one is interested in the complexity with respect to the output size as well: A problem is solvable in *output polynomial time* if there exists an algorithm whose running time is polynomial in the size of the input and the output.

This talk outlines some results from [1], where the complexity w.r.t. the size of the output of some multi-objective combinatorial optimization problems is investigated. They show, that the multi-objective shortest path problem with a given start and end node is not solvable in output polynomial time, unless $P = NP$. Further, they give a sufficient condition for solvability in output polynomial time.

References

[1] F. Bökler, M. Ehrgott, C. Morris, and P. Mutzel. Output-sensitive complexity of multiobjective combinatorial optimization. *arXiv preprint arXiv:1610.07204*, 2016.